Texturing 3D Geometries Using Reaction Diffusion

Hayden Devlin

School of Computer Science, University of Birmingham
msc97hfd@cs.bham.uk

Introduction

Designing surface textures with a natural appearance in manufacturing is a difficult and arduous task. This project presents a major step toward automating the generation of organic surface textures for arbitrary 3D geometries. Reaction Diffusion is presented as a powerful design tool, able to create an enormous variety of highly natural and aesthetic textures. Through the creation of a versatile new representation for Reaction Diffusion Systems the traditional system is adapted to achieve 2½D displacement texturing on the surface of any arbitrary 3D shape.

Reaction Diffusion

Reaction Diffusion Systems are a mathematical model describing the chemical interaction between two substances and their changing spatial distribution over time. In 1951 Alan Turing proposed that Reaction Diffusion occurring in organic tissue is responsible for morphogenesis [1] and demonstrated this by modelling a simple system based on two chemicals, $a$ and $b$, in continuous space. From an initial pair of partial differentials he derived two discrete numerical solutions to govern the behaviour of the two substances upon which a practical simulation can be built.

\[
\Delta a_i = S_a(16 - a_i b_i) + D_a(a_{i+1} + a_{i-1} - 2a_i)
\]
\[
\Delta b_i = S_b(a_i b_i - b_i - \beta) + D_b(b_{i+1} + b_{i-1} - 2b_i)
\]

Reaction Diffusion Equations for a 1D System

The first part of each equation denotes the reaction between the two chemicals and the second describes the diffusion of each. These equations function in a 1D space of points, but by simple extension of the diffusion Laplacian can allow it to function in $n$ dimensions [2].

\[
\text{Laplacian} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\]

Creating textures for simple 3D Shapes

The representation for applying Reaction Diffusion in 2D can be a simple 2D array of chemical values. Applying reaction diffusion over an $n$ dimensional space is simply a matter of changing the dimensionality of this array and the diffusion operator. It is not so obvious how to represent a Reaction Diffusion System to take place on the surface of a shape where the spatial distribution is more complex and neighbourhood sizes can vary.

Accounting for Varying Size Neighbourhoods

Initially only simple shapes that could be created from identical equilateral polygons were used so that the distances between neighbours would always be the same. This was beneficial as it is consistent with the standard model of Reaction Diffusion that uses a Laplacian diffusion matrix to calculate chemical diffusion. In order to account for the varying neighbourhood sizes brought about by the structure of the shape surface this diffusion operation had to be adapted.

The function causes equal diffusion into and out of the Cell and between all neighbours. Abstracting this from the spatial model, the diffusion calculation can be extended to allow for additional equidistant neighbours by scaling the amount of diffusion from each neighbour to sum to the amount taken from the centre point.

Creating Textures for Complex 3D Models

In order for the system to be really useful in practice it needs to be able to function on any 3D model. It is extremely rare for 3D model design to consist entirely of identical equilateral triangles. Accounting for inconsistent triangle dimensions required another adaption of the diffusion operator.

The ratio of diffusion into and out of a Cell is maintained, but instead of taking the chemical equally from the neighbouring cells it is taken inversely proportional to the distance between the cells. This means that more diffusion will occur between cells that are very close to each other, and less diffusion will occur between those neighbours that are further away.

Conclusion

With the Cell representation and the adaption of the diffusion operation, Reaction Diffusion has been successfully adapted to for displacement texturing of 3D surfaces to any 3D model. The results show the same patterning behaviour of standard Reaction Diffusion can be achieved on complex surfaces by changing the model of diffusion. By varying the Reaction Diffusion parameters an enormous variety of textures can be observed and its this new application to practical design shows its potential be an extremely useful tool in manufacturing.

References